

Fluency

Day 2

Video: $8 + 6$

- What are examples of how the ten-frame model supports student strategies for $8 + 6$?
- How does the ten-frame encourage students' understanding of 10 as an important number in computation?
- What seems to be a core strategy used throughout this number talk?
- $8 + 6$ is considered to be a basic addition fact. What role can number talks play in helping students build basic fact knowledge?



Reflection

- What is fluency? What new insights have you gained? How has your thinking changed?

Fluency

- *Efficiency* implies that the student does not get bogged down in many steps or lose track of the logic of the strategy. An efficient strategy is one that the student can carry out easily, keeping track of sub-problems and making use of intermediate results to solve the problem.
- *Accuracy* depends on several aspects of the problem-solving process, among them, careful recording, the knowledge of basic number combinations and other important number relationships, and concern for double-checking results.
- *Flexibility* requires the knowledge of more than one approach to solving a particular kind of problem. Students need to be flexible to be able to choose an appropriate strategy for the problem at hand and also to use one method to solve a problem and another method to double-check the results.

Subitizing: Fundamental Skill in Development of Number

- **Read the article “Subitizing: What is it? Why teach it?”**
- **Discuss with your group: Why is subitizing such an important skill?**
- **Are there any connections between subitizing and the content standards? If so, what are they?**

Learning Progression

What are learning progressions?

- Read a small part of the learning progression for Operations and Algebraic thinking.
- How does this part of the learning progression connect to fluency?

Understanding Addition and Subtraction

Address the big ideas that are at the center of math fact teaching:

1. Our number system is a system of patterns
2. The order of the terms does not change the sum (commutative property)
3. Addition and subtraction are inverse operations
4. Numbers are flexible. They can be broken apart to more easily perform an operation
 - Guide the types of questions that are posed during class discussion (pg 17)

Building Understanding While Focusing on Fluency

- Use problem contexts/real-life situations
 - **Make sure all four categories of problems are addressed**
- Use models to represent addition and subtraction
 - Number lines, manipulatives, ten frames, number charts
- Productive talk
- Classroom environment



Explore concepts through problems

- Problem based contexts builds an understanding of operations- conceptual understanding.
- Classifying Addition and Subtraction problems (the chart)

Word Problem Types- (In Your Binder)

TABLE 1. Common addition and subtraction situations.⁶

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	Addend Unknown	Both Addends Unknown ¹
Put Together/ Take Apart²	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare³	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$	("Version with *more*"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? ("Version with *fewer*"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$	("Version with *more*"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? ("Version with *fewer*"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$, $? + 3 = 5$

¹These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

²Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

³For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Word Problem Types

- Look at the word problems. Work with your table to classify them according to the common addition and subtraction word problems.
- Draw a representations for each problem (number line, picture, etc.) and write a number sentence to show how students might solve it.

Classroom Environment

- Discussion
- Partner Work
- Interactive bulletin boards
- Word walls
- Centers

Explore concepts through literature

- When reading or listening to stories, followed by discussions of the math situations represented in the stories, our students experience addition and subtraction in a context that makes sense to them.
 - *Rooster's Off to See the World*
 - *Animals on Board*
 - *Elevator Magic*
- To integrate the literature into math fact lessons, a “before, during, after” approach is recommended.

Possible Teaching Sequence

Mastering the Basic Math Facts in Addition and Subtraction

- $+1/+2$
- $+0$
- $+10$
- Doubles
- Making ten
- Using tens
- Using doubles

Looking at One Example: Plus 1 & Plus 2

What are the big ideas around adding with 1 & 2?

- The sum when 1 is added to a quantity is the next counting number.
- Our number system is a system of patterns.
- Addition is a joining or combining process.
- Subtraction is a separation or comparison process.
- The order of addends does not change the sum.
- Addition & subtraction are inverse operations.

Possible Questions to Support the Big Ideas of +1, +2

- When you add 1 (2), how do you know what the sum will be? Explain your thinking.
- Does the order of the addends change the sum? Give examples to justify your thinking.
- How might a number line (or number chart) help with +1 and +2 facts?
- How do you know when your answer makes sense?
- What is $30 + 1$? Explain how you know.
- What is a way to find a -1/-2 fact?

Literature Connection

- *Mouse Count* – read and discuss. Add cubes to a jar while reading story.
- *Mice in a Jar* activity. For each bag, count the number of mice. Then add 1 & record the result in a number sentence. Then add 2 mice & record. Find a way to organize your data.
- Debrief activity
 - What questions might you ask about the data collected?



Word Problems and Number Strips/Charts

- Students need to visualize the facts using a concrete model and move from concrete/visual experiences to symbolic representations. (C-R-A)

Word Problems and Number Strips/Charts

- Using the 20's Chart in your binder, represent each problem:
 - Katie took 5 bites of an apple. Then she took 1 more. How many bites did she take?
 - Kevin put 7 rocks in his wagon. Then he put 2 more rocks in his wagon. How many rocks did he put in his wagon?
 - If Kevin took two rocks out of his wagon, how many rocks would be in his wagon?

Word Problems and Number Strips/Charts

- Pose problem such as the following:
 - Allison had 5 lollipops and 1 peppermint. How many pieces of candy did she have?
 - Brendan had 1 lollipop and 5 peppermints. How many pieces of candy did he have?
 - Allison won 3 tickets at the arcade. Then she won 2 more. How many tickets did she win?
 - Brendan won 2 tickets at the arcade. Then he won 3 more. How many tickets did Brendan win at the arcade?

What is this series of tasks allowing students to develop?

Check for Understanding

- Jimmy says that $2 + 3$ has a different sum than $3 + 2$. Do you agree with him? Why or why not?
 - Why might you use a prompt such as this one?

$+1/+2$

- $+1/+2$ Spin
- Challenge students to summarize their thinking about $+1/+2$
 - Did the number strip help you add 1 or 2? How?
 - Do you need the number strip to add 1 or 2?
 - What if you did not have a number strip, how could you add 1 or 2?

Building Automaticity

- Short practice – daily routine
 - Fact card practice
 - Fact card jumps
 - Games
 - I Spy 1 More
 - Hop the Line
 - Dot Addition
 - Plus One Two Bingo

Connect to Subtraction

- Revisit *Mouse Count*
- Create 1 Less/2Less Story Problems
- Hop the Line Subtraction
- Less Than the Ten-frame
- Minus One, Minus Two Bingo
- What's in the Bag
- Dot Card Subtraction
- Spinning More or Less

Other Groups of Facts

- For the group of facts assigned to your group, give an overview similar to the one you just experienced for $+1/+2$.
 - $+0$
 - $+10$
 - Doubles
 - Making ten
 - Using tens
 - Using doubles

Retaining Math Facts

- Small amounts of practice with games and/or fact cards throughout the year. (See p. 141)
- Time to discuss how they know the answer to the fact, not just giving the answer.
- Use of concrete objects and word problems.

Timed Tests

- Read “Faster Isn’t Smarter: The Trap of Timed Tests.”
- Discuss the following:
 - What issues or challenges does this message raise for you? In what ways do you agree with or disagree with the main points of the message?



Planning

- Start to develop a plan that will help you increase your students' fact fluency during the coming school year.



Homework

- Read pages 1-26 of *Mastering the Basic Math Facts in Multiplication and Division*.